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Toughening of Cement and Other Brittle Solids with Fibres [and Discussion]

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Toughening of cement and other brittle solids with fibres

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In common with other brittle solids, cements are toughened much more by the incorporation of fibres than by inclusions of other geometries. The largest energies required to break a specimen are found when multiple fracture of the specimen occurs before final failure. Theoretical models of a crack moving normal to a set of parallel fibres will be considered, to show that the crack spacing and first cracking strain should depend on the area of fibre–matrix interface per unit volume of composite. The first cracking strain is shown to increase for all fibre volumes provided that the fibre spacing is less than the critical flaw size according to the Griffith's equation. The theoretical models are compared with experiment and the practical difficulties of defining first cracking strain and interfacial area mentioned. The best practical means of assessing the resistance to failure of the composite is the work done per unit volume of the specimen in separating it into two distinct pieces. The maximum values of toughness attainable – some $10^6\,\mathrm{J\,m^{-3}}$ – can decrease with time under external weathering, owing to continuing hydration of the matrix and consequent increase in the critical volume fraction of fibres.

1. Introduction

(*a*) *Brittle solids*

Brittle solids are those that break without large amounts of plastic flow, so that the total work of fracture measured in, say, a controlled notch bend test of the Tattersall & Tappin (1966) type is less than about $1000\,\mathrm{J\,m^{-2}}$ *and* the strain to failure is no more than 1 % or so.

Small increases in toughness can be obtained by introducing pores (Cooper 1977; Coppola & Bradt 1973); by the introduction of rubber particles (Kunz-Douglass *et al.* 1980); by introduction of particles of a plastically deformable metal (Stett & Fulrath 1968; Krstic & Nicholson 1981) or by inclusion of a material that can undergo a stress induced phase transformation (Claussen 1978). Although these methods produce a measure of increase, the largest values of work of fracture that can be obtained fall very far short of those that can be achieved by the introduction of long fibres (see, for example, Phillips (1983) for a review).

These large increases are always accompanied by the appearance of the phenomenon of multiple fracture (Aveston *et al.* 1971). When multiple fracture occurs a series of more or less parallel cracks are formed in a body, which run in a direction approximately normal to the major principal axis of strain. The appearance of these cracks and their running completely across the body does not lead to failure in the sense of separation into two distinct pieces (although one component has certainly broken) and considerable extensions of the body are still possible in a direction normal to these cracks.

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It is important then to separate two quite distinct processes, which are:

(*a*) the (rapid) propagation of the first set of parallel cracks across the body, which starts at a more or less well defined strain, approximating that of the failure strain of the more brittle component tested separately and

(*b*) the subsequent process of absorption of energy in deforming the body to failure. This depends principally upon the properties of the high elongation component modified by the prior occurrence of (*a*).

Since the strains at which (*a*) and (*b*) occur are in general widely different and the total work of fracture is governed by (*b*), it is obvious that conventional fracture mechanics has no applicability in assessing the resistance to fracture of the body as a whole. Under these conditions the resistance to fracture is best, if crudely, assessed by quoting the work done on the body per unit volume in separating the specimen into two halves. This work per unit volume is the area under the stress–strain curve (Aveston & Kelly 1980). We give some examples for cement matrix materials in §§4, 5 below.

The rapid propagation of the first set of cracks, which traverse one of the phases, leaving the other essentially undamaged, may, however, be governed by some modified form of fracture mechanics, and this we discuss in §§2, 3. Before doing so, it is worth emphasizing that in the literature concerning fibre reinforced cements, a clear distinction is not always drawn between processes (*a*) and (*b*), which leads to confusion. If bend tests are done, process (*a*) may occur on a rising load–deflexion curve and a series of parallel cracks will traverse the tension surface of the specimen and then proceed to penetrate towards the interior. Irreversible work is done in this process and the use of the load–centre point deflexion curve, with the idea of the J integral, to obtain a measure of the potential energy available to crack the material (for example, by following the beautifully clear procedure of Begley & Landes 1972*a*, *b*) is in our opinion quite inadmissible.

The fracture of a monolithic piece of a brittle solid is usually rationalized nowadays by supposing it contains, either at the surface or in the interior, fissures or flaws of varying length, figure 1 (*a*). The propagation of the largest of these flaws, which is oriented normal to the applied stress, gives the breaking stress according to the formulation of Griffith (1920).

The addition of fibres to such a body may be assumed to affect the initial flaw size, limiting the length of flaws to the inter-fibre spacing, as in figure 1 (*b*); or leaving it unaltered, and hence with fibres traversing these cracks, figure 1 (*c*). If the process of introduction of fibres greatly alters the flaw distribution in the matrix then a discontinuous change in cracking strain of the matrix is expected to be observed on introducing fibres and there should be little connection between the cracking strain of the matrix without fibre and that observed when fibres are introduced. Alternatively, if the situation is that shown in figure 1 (*c*) a smooth increase in first cracking strain with introduction of fibres is expected.

Although there are many experiments that indicate an increase in first cracking strain with introduction of a high elongation phase into a brittle matrix or brittle laminate system, experiments with rather small volume fractions of the high elongation phase have usually been confined to cements (Aveston *et al.* 1974; Hughes 1983) and the nature of the initial flaw in cements needs a little discussion, which we give in §1 (*b*). Although there is rather smooth variation in first cracking strain with increase in volume fraction of the high elongation phase – which supports the model in figure 1 (*c*) – unfortunately, accurate measurements in tension of the cracking strain of the unreinforced matrix are exceedingly difficult.

Independently of whether the schema of figure 1 (*b*) or of figure 1 (*c*) apply in the initial stages of deformation, there is no doubt that the final stages can be described by a diagram like figure 1 (*d*). The condition for this to occur is that the high elongation phase is able to bear the total load.

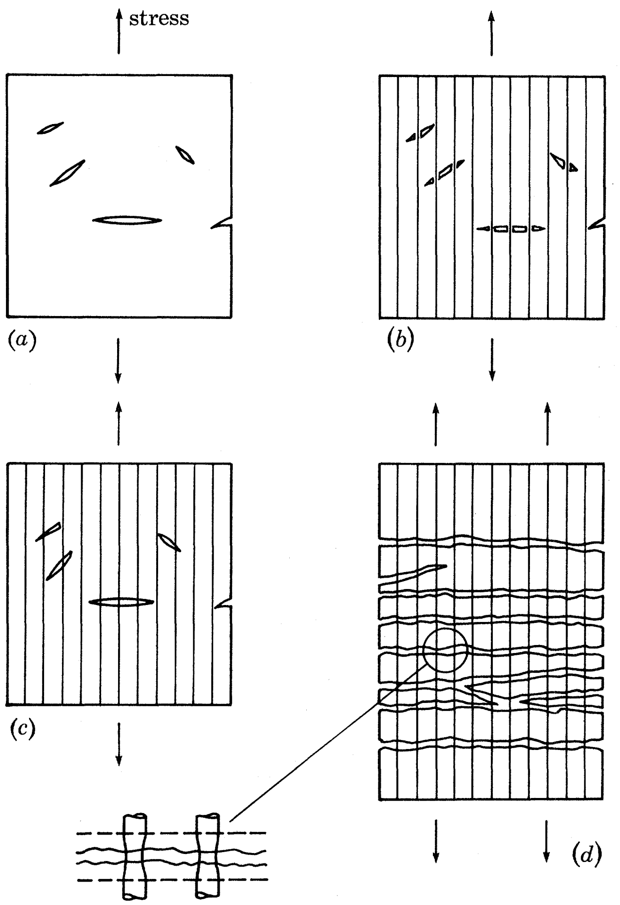

FIGURE 1. (*a*) Schematic representation of flaws in a brittle solid. (*b*) Schematic representation of the same solid, but now containing fibres, indicating that the initial flaw distribution is influenced by the introduction of fibres. (*c*) This is the same as (*a*), but indicates that the initial flaw distribution is not influenced by the introduction of fibres but that the fibres straddle the initial flaws, $s < 2c$ (see text). (*d*) Schematic representation of a cement paste that has undergone multiple fracture of the matrix (cement). The inset illustrates that the longitudinal stress in the fibres is not uniform.

The condition is

$$V_f \geqslant \sigma_{mu}/(\sigma_{fu}+\sigma_{mu}-\sigma'_f), \tag{1}$$

which, if both components are linearly elastic to the first cracking strain, can be written as

$$V_f \geqslant E_c \epsilon_{mu}/\sigma_{fu}, \tag{2}$$

where V is the volume fraction, E the elastic modulus, σ_u the failure stress, ϵ_u the failure strain, σ' the stress on the high elongation component when the other fails, and the subscripts m, f and c refer to the matrix, high elongation component and composite respectively.

If the volume fraction of high elongation phase is constant throughout the specimen and the

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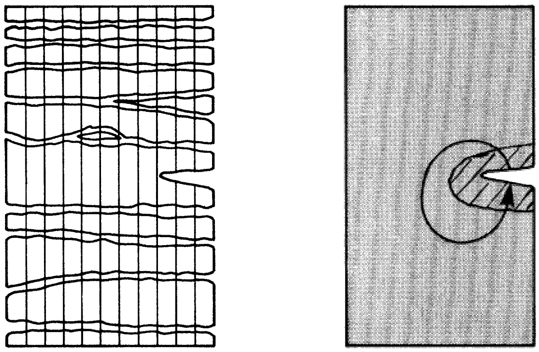
interfacial conditions are also constant, then there is a lower limit to the spacing of the parallel cracks in figure 1 (*d*) given by

$$x = \mathscr{E}_{\mathrm{mu}} \frac{V_{\mathrm{m}}}{V_{\mathrm{f}}} \frac{r}{2\tau} = \frac{\sigma_{\mathrm{mu}} V_{\mathrm{m}}}{\tau \beta}, \qquad (3)$$

where β is the interfacial area per unit volume of composite (equal to $2V_{\mathrm{f}}/r$ for aligned fibres of circular cross section) and τ is a shear stress.



(*a*) (*b*)

FIGURE 2. (*a*) A notched specimen containing fibres, which has undergone multiple fracture. (*b*) Model of a deformed notched specimen that is usually considered when one assesses fracture resistance with the *J*-integral.

If a specimen such as that in figure 1 (*d*) contained an artificially introduced notch or crack, which had been introduced by severing the matrix and high elongation phase, then its appearance would be as in figure 2 (*a*). Figure 2 (*a*) may be compared with figure 2 (*b*), which represents a specimen containing no high elongation phase. In bringing the specimen in figure 2 (*a*) to the state of multiple internal cracking with the consequent non-uniform elongation of the high elongation phase across the cracks, potential energy of the loading device has been used to fracture the matrix and cause *irreversible* sliding friction at the interface between the phases and remote from the notch, and so it is not all available to drive the notch further by fracture of the high elongation phase. The path for the J integral in figure 2 (*b*) cannot even be outlined on figure 2 (*a*).

(*b*) *Initial microstructure of cements*

There is currently, of course, as shown by this symposium, a good deal of interest in the elucidation of the microstructure of Portland cement paste. For the purpose of understanding the initial cracking strain and the work of fracture of the material we wish to emphasize two features. First, that the tensile stress–strain curve of the material is nonlinear, showing a decreasing slope after strains of less than 10^{-4}. A small permanent set may be observed on unloading from any stress, even one as low as 1 MPa. Part of this set at *room temperature* can be attributed to creep. These characteristics are shared with paper (see, for example, Seth & Page 1980) and cast iron (Haenny & Zambelli 1983). They are characteristic of a material that contains a large number of internal flaws and fissures that change their shape irreversibly under an applied load.

The breaking strength of such materials is found to be insensitive to the presence of 'small' notches cut into the surface because these notches are often 'shorter' or smaller in one dimension than the internal flaws already present in the material. Only if notches longer than a certain minimum size are cut does the strength depend upon their presence. For conventional cement paste this minimum size is about a millimetre (Higgins & Bailey 1976; Birchall *et al.* 1981).

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The microstructure of a fibre reinforced cement must be thought of then as depicted in figure 3, where the matrix is of a more or less granular nature, consisting of partly hydrated cement grains bound together by a gel, with the whole containing voids that may be of size up to 1 mm, though more frequently of diameter 100 μm. There may, in addition, be long thin fissures of a crack-like

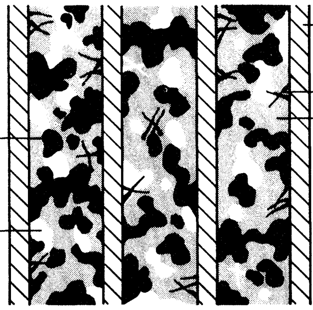

FIGURE 3. Schematic illustration of a cement matrix containing fibres. There are at least four phases present: fibres, unhydrated cement grains, hydration products and voids.

nature. The apparently smooth, well compacted surface of fibre reinforced cement cannot be regarded as really typical of the microstructure of the interior. It is very likely that there is no real adhesion between fibres and matrix and, as illustrated in figure 3, the interface will be characterized by more or less intermittent contact. Values derived for 'bond strengths' are then very uncertain (Laws 1982), and this is particularly so for polyalkene fibres in cement, which we shall be considering in §§3, 5. With polyalkene fibres, in addition, the values found for τ (cf. equation (3)) are extremely small, usually less than 0.5 MPa; an order of magnitude less than observed in other systems.

2. First cracking strain in the matrix

(a) *Without fibres*

If the matrix has a normally observed cracking strain, with no fibres present, of mean value ϵ_{mu}, then this can be interpreted to mean that it contains cracks of which the largest lying normal to the applied stress has a length $2c$, where $2c$ is given by the Griffith relation

$$\sigma/E = [g/E\pi(1-\nu^2)\,c]^{\frac{1}{2}} = \epsilon_{mu}, \tag{4}$$

where g replaces twice the surface energy in the original Griffith formulation, and a condition of plane strain is assumed.

This result can be derived (Friedel 1959; also see Hirth & Lothe (1968)) by considering an elastic equilibrium crack of which the length is $2c$ and the maximum opening B, so that B and c are related by the equation

$$\sigma/E = B/\pi(1-\nu^2)\,c. \tag{5}$$

If such a crack is to advance in a material that has a resistance to crack advance, g, then the applied stress acting through a distance equal to the maximum displacement, i.e. B, must do work at least equal to the resistance per unit area g. Hence, we must have

$$\sigma B \geqslant g. \tag{6}$$

By eliminating B between equations (5) and (6) we recover (4).

Thus at a critical crack opening B, the rate of release of strain energy exceeds the resistance to crack growth and the crack spreads in an unstable fashion.

If we apply equations (4) and (5) to Portland cement, taking $\epsilon_{mu} = 2 \times 10^{-4}$, $E = 30$ GPa, $g = 5\,\mathrm{J\,m^{-2}}$ and $\nu = 0.25$, we find that the largest cracks present in the material have a length $2c \approx 2.8$ mm and a critical opening of 0.8 μm.

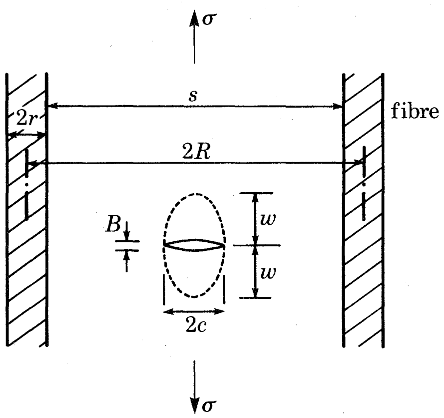

FIGURE 4. A crack under stress in a material with a fibre spacing much greater than the initial flaw size (see text for nomenclature).

(*b*) *Fibre spacing* '*s*' $\gg 2c$

When there are no fibres present, the matrix fails at an average strain given by equation (4). Assuming that the critical width for the flaw shown in figure 4 is B and that the matrix strain increases linearly from zero at the flaw to ϵ_{mu} at distance w, then the maximum relaxation distance w on each side of the crack may be found from

$$B = 2w \times \epsilon_{mu}/2 = w\epsilon_{mu}. \tag{7}$$

Substituting in equation (5) gives

$$w = \pi(1-\nu^2)\,c \simeq 3c. \tag{8}$$

Figure 4 shows the crack under stress, i.e. open, with width of opening B.

If fibres are introduced with a surface to surface spacing s, much greater than $2c$, see figure 4, the crack will advance unstably in the matrix when equation (4) is obeyed; with E equal to the modulus of the composite, before it encounters the fibres. When the unstable crack eventually meets the fibres, all the processes described by Aveston *et al.* (1971) are expected to occur provided that equation (1) is obeyed. If equation (1) is not obeyed and so the specimen breaks into two halves, there will be in principle a small increase in g if measured after total fracture (for example in a Tattersal & Tappin (1966) test), compared with the value of g for the matrix alone because of the need to deform the fibres to fracture.

An interesting prediction from equation (4) is that if the fibres added are of lower modulus than that of the matrix, then the cracking strain may actually be increased because E in equation (4) will be less than that of the matrix. It is important to determine for practical fibre reinforced systems over what range of volume fraction (V_f) equation (4) applies, i.e. whether s, the fibre spacing, is greater or less than $2c$.

(*c*) *Fibre spacing s in relation to 2c*

The surface to surface separation of the fibres is given by

$$s = 2(R-r) = 2r[(\alpha'/V_f)^{\frac{1}{2}} - 1], \tag{9}$$

where $\alpha' = \pi/2\sqrt{3} = 0.912$ for a hexagonal array of fibres and $\alpha' = \pi/4 = 0.785$ for a square array, R is the fibre separation (centre to centre) and r the fibre radius.

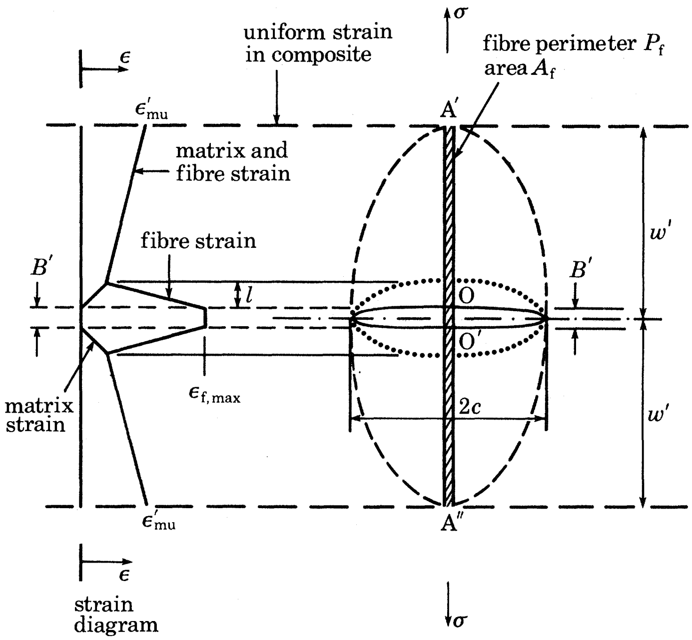

FIGURE 5. A possible zone of relaxation of strain around a crack under stress that is traversed by a single fibre. The left side of the figure illustrates the variation in strain within the fibre and the matrix along the line A′A″.

There will be at least one fibre within the critical crack length for the unreinforced matrix, if $2c$ equals s. So from equations (4) and (9)

$$2g/E_c\pi(1-\nu^2)\,\epsilon^2_{mu} = 2r[(\alpha'/V_f)^{\frac{1}{2}} - 1]. \tag{10}$$

By taking values appropriate to a high strength Portland cement at w/c ratio = 0.2 and $E_c = E_m = 30\,\mathrm{GN\,m^{-2}}$, $g = 15\,\mathrm{J\,m^{-2}}$, $\nu = 0.25$, so $(1-\nu^2) \approx 1$, $\epsilon_{mu} = 4 \times 10^{-4}$ (0.04%) and a radius of fibre 50 μm, then equation (10) is satisfied for a volume fraction of fibre (V_f) greater than about 0.2%. The fibre spacing is then between 1 and 2 mm and so the argument is consistent with that given in § 1 (*b*).

For practical fibre cements containing asbestos fibres, glass fibre bundles, or polypropylene films at effective fibre volumes of 1.5 %–10 % the situation we must consider is therefore one in which $s < 2c$. Where steel fibres are concerned with r about 0.25 mm, effective fibre volumes of 3 % would be required before $s \leqslant 2c$ and the lower fibre volumes *ca.* 1 % used in practice would effectively rule out an increase in the applied strain before the cracks become unstable in the matrix.

For comparison, a hot pressed pyrex glass with $E = 70\,\mathrm{GN\,m^{-2}}$, $\epsilon_{mu} = 0.14\,\%$ and $g = 10\,\mathrm{J\,m^{-2}}$, containing carbon fibres of diameter 8 μm requires a V_f equal to about 1.7 % before $s \leqslant 2c$. The volume fractions normally used exceed this figure (Phillips *et al.* 1972) and so we expect fibres to straddle the initial Griffith flaws. On the other hand, for a glass of the same properties, containing silicon carbide fibres of 140 μm diameter (Prewo & Brennan 1980), a volume fraction of greater than 40 % is needed before $s \leqslant 2c$, whereas one of the volume fractions they used was 35 %, fibres are therefore not expected to straddle the initial cracks.

(d) *Fibre spacing $s \leqslant 2c$*

When the fibre spacing is less than the Griffith flaw size we assume that the introduction of fibres does not by itself alter the length of the initial flaws in the matrix. We model the situation in figure 5, which for clarity shows the crack in the matrix as partially open. To expose the argument we suppose the crack to be traversed by a single fibre.

Imagine the strain in the specimen to be increased from zero with the 'crack' initially closed. At O and O′ the shear stresses between fibre and matrix will be rather large and if we assume that this shear stress can be described by a quantity τ, which is a constant independent of distance along OA′ and O′A″, then we expect the matrix to relax by sliding back over the fibres. The fact that the matrix has to slide back over the fibres means that the strain in the matrix will increase more rapidly along the lines OA′, O′A″, than it would have done in the absence of the fibres.

If the matrix slides back over a distance l a force $2\pi r\tau l$ is applied to a circular sectioned fibre. This produces a strain in the fibre of $\Delta\epsilon$, and, if we assume that the opening of the crack B' is solely due to this differential slip then

$$l = B'/\Delta\epsilon. \tag{11}$$

We also have

$$E_f\Delta\epsilon = 2\pi r\tau l/\pi r^2 = 2\tau l/r. \tag{12}$$

So, by eliminating l, the stress in the fibre $\sigma_{f,\,max}$ cannot be less than $E_f\Delta\epsilon$ or

$$\sigma_{f,\,max} \geqslant (2\tau B' E_f/r)^{\frac{1}{2}}. \tag{13}$$

For $B' \approx 1\,\mu m$, $\tau = 3\,MPa$, $r = 5\,\mu m$, $E_f = 70\,GPa$ (glass), $\sigma_{f,\,max} = 290\,MPa$, which is much greater than 28 MPa, the value of $E_f\epsilon_{mu}$ for $\epsilon_{mu} = 4\times10^{-4}$.

We sketch the distribution of strain in fibre and matrix on the left side of figure 5. This sketch is made for a linear variation of strain with distance. We do not know that this variation is linear in reality of course, but it must be true that the fibre and matrix differ greatly in strain at the crack face and that the variation in each as one moves away from the crack is of opposite sense. Also, the fibre and matrix must attain the same strain before the edge of the relaxed region (within which the strain is less than the average strain of the specimen as a whole) is reached.

The fibres thus exert a closing force on the crack and reduce its opening. If there are N fibres crossing a crack, the total closing force, F, reducing the opening of the crack is

$$F \approx N\sigma_{f,\,max}A_f. \tag{14}$$

The quantity NA_f is proportional to the volume fraction of fibres. Hence we expect a continually increasing curve (from $V_f = 0$) to relate fibre volume fraction with the matrix strain at which unstable crack propagation occurs, and it must be borne in mind that this is so whether or not the critical volume fraction given by equation (1) is exceeded.

3. Theory of the increased matrix failure strain

A theory of the increased failure strain of the matrix has been given by Aveston *et al.* (1971) and developed a little further by Kelly (1976). It applies when the strain energy released by a growing crack becomes independent of crack length. This applies when many fibres cross a crack and the quantity $\tau\beta$ is large. The theory will in general give a lower limit to the strain that must be exceeded for cracking to occur. Other attempts to explain the increased cracking strain have been given by Romualdi & Batson (1963), Tardiff (1973), Spurrier & Luxmore (1973, 1976).

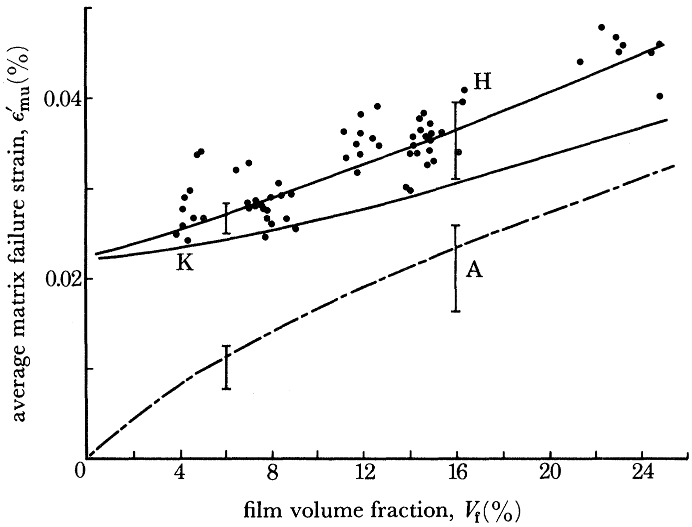

FIGURE 6. Relation between average matrix cracking strain and film volume fraction of a composite containing fibrillated polypropylene. (Effective fibre radius 23 μm, fibre modulus 7.7 GPa, matrix modulus 31.5 GPa, bond strength (τ) 0.5 MPa, matrix work of fracture 5 J m^{-2}, failure strain of unreinforced matrix 0.0222 %, Griffith crack length 4.1 mm.) Upper and lower bars at V_f = 6 % and 16 % indicate the variation of failure strain predicted for values of τ between 0.2 MPa and 0.8 MPa. H, Hughes (1983); K, Korczynskyj *et al.* (1981); A, Aveston *et al.* (1971).

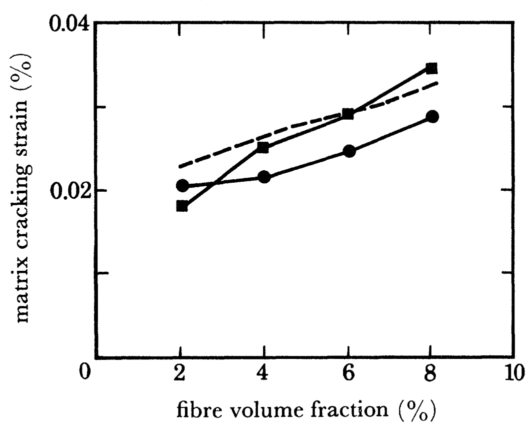

FIGURE 7. Theoretical and experimental values of enhanced matrix failure strain of cement that contains glass fibres randomly oriented in two dimensions. Experimental points after Ali *et al.* (1975). Theoretical curve from Hughes (1983). The assumed efficiency factor was 0.27. Squares represent data for specimens stored in water and dots are for specimens stored in air.

A computer calculation based on the idea of modelling a growing crack by assuming an initial shape for the crack and expanding it has been given by Korczynskyj *et al.* (1981). We set up a similar computer programme to theirs to see whether their theory applied to the results of Hughes (1983), who has made a very large number of measurements of the initial cracking strain for bars of cements containing continuous lengths of fibrillated polypropylene and polyethylene film. We believe that the assumptions of Korczynskyj *et al.* (1981) are not consistent with the occurrence of multiple fracture showing the experimental dependence on fibre volume fraction and the dimensions of the fibres and we therefore developed an alternative theory based on somewhat different assumptions. It is outlined in the Appendix, which is derived from Hughes (1983).

Theoretical predictions of the matrix failure strain are compared with experiment in figure 6, which contains Hughes's results. Figure 7 compares previously published results for glass fibre reinforced cement (Ali *et al.* 1975) with the predictions. Before commenting on these figures, it should be recognized that the Aveston *et al.* (1971) model (A.C.K. model) always gives a lower limit to the first cracking strain. This strain must be exceeded for cracking to occur and experimental results show the limit to be a valid one. However, as shown in §2(*d*), fibres arranged parallel to the applied stress and crossing a crack always hinder its growth and hence the presence of such fibres is always expected to increase the cracking strain of the matrix at all V_f when $s < 2c$, even if V_f is very small.

In figure 6, measurements of the matrix cracking strain from A.C.K. theory in the form

$$\epsilon'_{mu} = \left[\frac{6g_m \tau E_f V_f^2}{E_c E_m^2 r V_m}\right]^{\frac{1}{3}}, \tag{15}$$

the theory of Korczynskyj *et al.* (1981), and the theory from the Appendix are compared with experiment for specimens of fine grained ordinary Portland cement mortar containing between 4 % and 26 % (by volume) of continuous fibrillated polypropylene film. The parameters used to obtain the curves are given in the figure description. The difficulties of measuring the quantities, τ, r (effective radius of fibre), V_f and the moduli are given in detail in Hughes (1983) and will be published elsewhere. The value of the bond strength, τ, in equation (12), which applies to the initial fracture of the matrix, is particularly difficult to determine experimentally. Hence, the variation in calculated average matrix failure strain is also shown in figure 6 for bond strengths of 0.2 MPa and 0.8 MPa, which encompass the range of values calculated from equation (3) with data from tensile multiple cracking experiments. The computer model given in the Appendix gives good agreement with experiment. As emphasized in the Appendix, the quantity L_2 (see figure A 1), which is a measure of the length of the relaxation zone on either side of the crack, is decreased with increase in β. It is this decrease that allows the model to be more consistent with the occurrence of multiple fracture than that of Korczynskyj *et al.* (1981). Quantitatively there is a problem, however, in that at 25 % fibre volume fraction we predict a length of relaxation of 3.5 mm on either side of a crack, whereas the crack spacing observed is a good deal smaller than this, typically less than 0.5 mm.

It should be borne in mind that a mere comparison with experiment of the theory of the variation of measured cracking strain with V_f is not a very stringent test of theory if that theory is scaled to pass through the observed unreinforced matrix cracking strain, as are both computer calculations. This is because the experimental scatter in cracking strain is 25 %–30 % and the increase in observed average value, even at volume fractions of 25 %, is less than a factor of two. Almost any theory that predicts an increase of ϵ_{mu} with V_f and passes through the value of ϵ_{mu} at $V_f = 0$ will be close to the experimental points.

The computer model given in the Appendix also accounts well for the experimental results on the increase of first cracking strain for glass reinforced cement reported by Ali (1975); see figure 7. In addition, the theory predicts, in accord with experiment, that the ultimate failure strain of the asbestos fibres in asbestos cement at $V_f = 0.05$ and 0.10, is reached before the crack becomes unstable. We therefore expect breaking of the specimen and first cracking of the matrix to occur simultaneously at a strain of 8.6×10^{-4} for $V_f = 0.05$, and at 1.2×10^{-3} for $V_f = 0.10$. These figures may not be directly compared with experiment because of the large volume of porosity in asbestos cement (Allen 1971) and because the fibres are not aligned. However, the prediction is in excellent accord with experiment.

4. Assessment of post-cracking toughness

Most of the energy absorbed in the complete fracture of fibre cements and concretes occurs after the matrix has cracked and can be calculated (Hibbert & Hannant 1982*a*). When fibre fracture occurs, the energy absorbed depends on whether the fibre volume fraction is greater or less than the critical value ($V_{f,\,crit}$) given by equation (2).

The area under the tensile stress–strain curve when expressed as energy per unit volume of material is in our opinion the most appropriate parameter to be used in design for energy-absorbing capability.† However, to compare results obtained at different laboratories and to calculate total energies required for failure, we recommend that all workers give the dimensions of their specimens so that both energy per unit volume and energy per unit area of final fracture surface can be calculated. The work done on the specimen in taking it through the complete stress–strain curve, when expressed as energy per unit volume, is given by

$$U = 0.5\,E_c\,\epsilon_{mu}^2 \quad \text{for} \quad V_f \leqslant V_{f,\,crit}, \tag{16}$$

$$U = 0.5\sigma_{fu}\,\epsilon_{fu}\,V_f + 0.159\,\alpha\,E_c\,\epsilon_{mu}^2 \tag{17}$$

for

$$V_f \geqslant V_{f,\,crit} \quad \text{and} \quad \alpha = E_m V_m / E_f V_f,$$

where, for fibres elastic to failure, the first term in equation (17) represents an upper bound to the strain energy required to strain the fibres to failure and the second term represents the minimum contribution from the multiple cracking of the matrix. The energy absorbed by multiple cracking is usually a relatively small proportion of the total energy.

5. Time dependent effects

Cements differ from most other brittle solids in that they show significant changes in properties with time under external weathering conditions although there is little change with indoor storage. Further, the fibre strengths may change with time in some composites. On economic grounds in practical composites, other than asbestos cement, the fibre volume is often kept to a minimum but care is taken to ensure that the critical fibre volume ($V_{f,\,crit}$) is exceeded so that multiple cracking occurs with a rising post-cracking tensile stress–strain curve. However, because of matrix changes under external weathering, and in particular an increase in E_m, the necessary critical fibre volume may increase substantially with time and if allowance is not made for this increase at the design and production stage, a composite that is initially tough and ductile with many cracks, may change its character to a brittle failure with a single crack.

† The energy absorbed before the component separates into two pieces is of importance when assessing the resistance of a structure to transient overloads such as random impact, wind loading, or explosion. It may not be related to criteria for acceptance under other conditions.

(a) *Hypothetical example*

To illustrate the effect that changes in the properties of the matrix can have on the energy absorbed to complete failure, we give an example within the range of practical composites. The properties of the matrix of a glass reinforced cement may alter with natural weathering to give a change in composite modulus (E_c) from about 22.5 GPa at 28 days to about 28.5 GPa after 5 years and 10 years (Building Research Establishment 1979). The matrix failure strain ϵ_{mu} at 28 days may be calculated from the bend-over point (i.e. where the stress–strain curve shows its first rapid change in slope) to be 4.2×10^{-4}, but it cannot be re-assessed after 10 years because of problems of testing. We therefore assume that it remains constant. Taking a value of fibre strength $\sigma_{fu} = 1000$ MPa and $E_f = 70$ GPa gives $V_{f,\,crit}$ from equation (2) equal to 0.95 % at 28 days and 1.2 % at 10 years. 1 % (by volume) of continuous aligned glass fibres exceeds $V_{f,\,crit}$ at 28 days and the energy, U, absorbed to complete failure will be 90 kJ m^{-3} from equation (17). After 10 years, however, V_f will be less than $V_{f,\,crit}$ and the energy absorbed will be only 2.5 kJ m^{-3} according to equation (16).

It is worth emphasizing that in this example both the glass content and strength are unchanged. The only changes that we postulated to have occurred are that the matrix has become stiffer and stronger and single fracture has replaced multiple fracture.

(b) *Experimental data for polypropylene films in cement*

We have obtained data on changes in elastic modulus and strain at the bend-over point (ϵ_{mu}) after 3 years natural weathering of composites containing 6 % (by volume) networks of fibrillated polypropylene film aligned with the stress. The data are:

$$28\text{ days},\quad E_c = 28\text{ GPa},\quad \epsilon_{mu} = 3 \times 10^{-4},\quad \sigma_{fu} = 280\text{ MPa};$$
$$3\text{ years},\quad E_c = 35.2\text{ GPa},\quad \epsilon_{mu} = 3.4 \times 10^{-4},\quad \sigma_{fu} = 270\text{ MPa}.$$

The critical fibre volume calculated from equation (2) increases from 3 % after 28 days to 4.4 % after 3 years. In this particular composite (Hannant & Keer 1983), where $E_f \approx 2.9$ GPa at strains greater than 1 % or so, the measured energy absorbed after 28 days was 790 kJ m^{-3} compared with a theoretical value (equation (17)) of 880 kJ m^{-3}. After 3 years the measured energy absorbed was 730 kJ m^{-3} compared with a theoretically predicted value of 890 kJ m^{-3}. Similar composites when stored under water for one year retain energies to failure in excess of 1 MJ m^{-3} (Hibbert & Hannant 1982*b*).

More recently produced polypropylene films have strengths of 600 MPa (Vittone *et al.* 1982) and these would require an increase in critical volume from 1.4 % at 28 days to 2 % at 3 years to maintain the high degree of toughness of the composite.

(c) *Glass fibre cement*

For glass reinforced cement, the following values have been published (Building Research Establishment 1979): 28 days, $E_c = 22.5$ GPa with a bend-over point at 9.5 MPa, which gives $\epsilon_{mu} = 4.22 \times 10^{-4}$, $\sigma_{fu} = 1000$ MPa; 10 years, $E_c = 28.5$ GPa, no accurate value for the bend-over point is given. If we assume no change in ϵ_{mu} and take σ_{fu} equal to 600 MPa after 10 years – for the recently developed fibre described by Proctor *et al.* (1982) – equation (2) gives an increase in $V_{f,\,crit}$ from 1 % at 28 days to 2 % at 10 years for aligned fibres.

Most glass reinforced cement is sprayed with short fibres in a random two-dimensional (2-D) orientation. To take account of the non-alignment, we multiply the value of σ_{fu} by 0.27 (the efficiency factor for stress, Oakley & Proctor (1975)) and find that a total fibre volume fraction of more than 3.5% at 28 days and more than 7.4% at 10 years is required to maintain ductility. Typical total fibre volumes are about 4% and it is known that the energy absorbed to failure, found from the area under the measured stress–strain curve (Majumdar & Laws, 1979), reduces from about 120 kJ m^{-3} at 28 days to less than 5 kJ m^{-3} at 5 years. This change may therefore be explained by the increase in critical fibre volume fraction with time.

Fibre concretes with less than the critical volume of short fibre at all ages, such as steel fibre concrete, may not suffer from such time dependent effects provided that fibre pull-out always remains the failure mechanism, although, the volume of specimen deformed to failure will be smaller if equation (2) is not obeyed.

Appendix

The model of crack growth developed by Korczynskyj *et al.* (1981) is based upon the assumption that the presence of fibres does not alter the length of the critical flaw or the size and shape of the relaxation zone around the flaw. Because the overall increase in length of the fibres within the zone (L_1 to L_2 in their figure 2) must be the same as that in the absence of the flaw, a proportion of the relaxed zone is at a constant strain, below that of the majority of the material. In §2(*d*) it was suggested that the maximum opening of the flaw would be reduced in the presence of fibres, owing to the closing forces they exert. However, Korczynskyj *et al.* fail to predict any change in the flaw opening at fibre contents up to 25% (by volume).

A new computer model has been described (Hughes 1983) that does not assume a constant size of the relaxation zone. This model is summarized here. In common with Korczynskyj *et al.*, the relaxation zone around a flaw in the unreinforced matrix is assumed to be elliptical with major axis three times the flaw length and of the form $x^2/9+y^2=c^2$, where $2c$ is the length of the flaw and the directions of x and y are shown on figure A 1. For the unreinforced matrix only the 'outer edge' of the ellipse is defined by a distance L_3 from the face of the flaw. The strain is assumed to increase linearly from zero at the face of the flaw to ϵ_m, the strain in the bulk of the material, at the edge of the ellipse, i.e. at a rate ϵ_m/L_3. The quantity L_3 does not define a distance from the crack in the *reinforced* matrix. This is the principal difference between the treatment here and that of Korczynskyj *et al.* (1981).

The presence of fibres is assumed not to alter the length of the critical flaw but alters the strain distribution and size of the relaxation zone as shown in figure A 1, which illustrates the situation immediately before unstable propagation of the flaw. Relative slip between fibre and matrix occurs for a distance L_1 from the face of the flaw. The strain in fibre and matrix is equal between L_1 and L_2 and is assumed to increase at a rate of ϵ'_{mu}/L_3.

The rate of change of strain in the matrix and fibre up to distances L_1 from the crack face is given by

$$\frac{d\epsilon_m}{dx}=\frac{2\tau V_f}{E_m V_m r}+\frac{\epsilon'_{mu}}{L_3}, \tag{A 1}$$

$$\frac{d\epsilon_f}{dx}=-\frac{2\tau}{E_f r} \tag{A 2}$$

respectively.

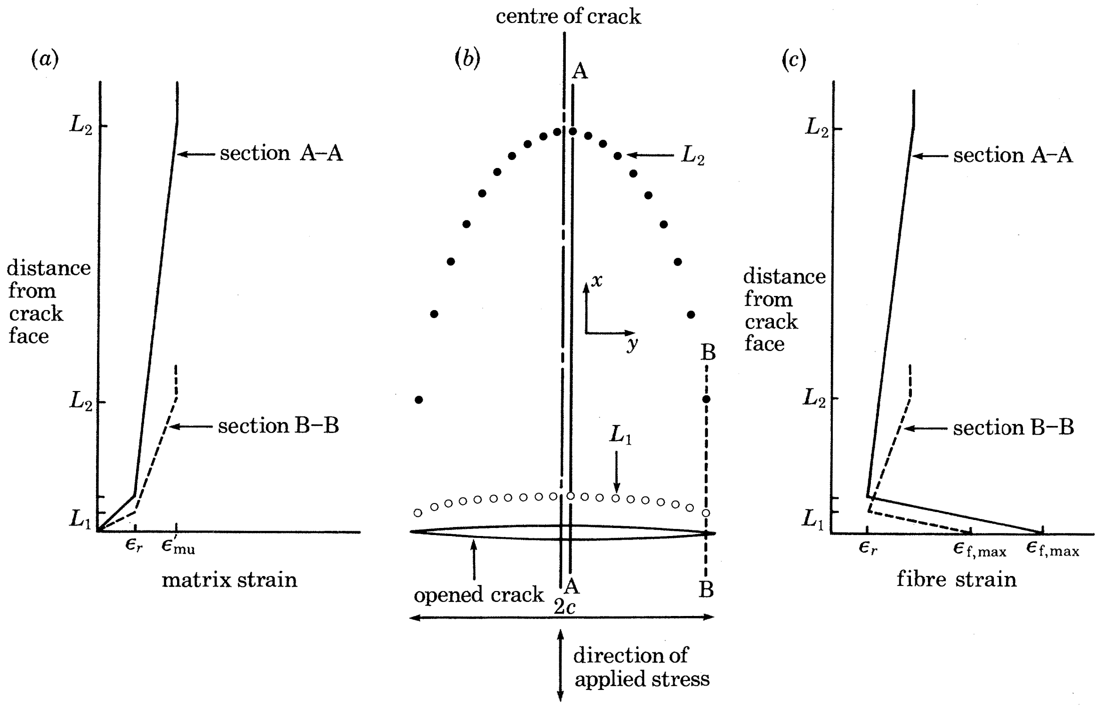

FIGURE A 1. Assumed distribution of strain in fibre and matrix along sections perpendicular to the crack face for a composite with a crack opened due to applied stress, fibre and matrix strains not to the same scale. L_2 (max.) $\equiv w'$ in figure 5 and also L_1 (max.) $\equiv l$ in figure 5. The strain, ϵ_r, at a distance L_1 is the same in fibre and matrix at a fixed value of y.

The strain in the matrix increases from zero to the average remote strain ϵ'_{mu}, figure A 1 (*a*). The strain in the fibre goes through a minimum at a distance L_1. The strain at a distance L_1 is $L_1 \mathrm{d}\epsilon_m/\mathrm{d}x$. The maximum fibre strain, $\epsilon_{f,\,max}$ is given thus:

$$\epsilon_{f,\,max} = L_1\left(\frac{\mathrm{d}\epsilon_f}{\mathrm{d}x}+\frac{\mathrm{d}\epsilon_m}{\mathrm{d}x}\right), \tag{A 3}$$

see figure A 1 (*c*). We also assume that

$$\epsilon_{f,\,max} = B''/L_1, \tag{A 4}$$

where B'' is the flaw opening at any distance 'y' from the centre of the flaw to the plane considered. Because of equations (A 1–A 4), B'' will vary with fibre volume fraction, which is different from the analysis of Korczynskyj *et al.* (1981).

At a distance L_1, the strains in fibre and matrix become equal to ϵ_r, such that

$$\epsilon_r = L_1 \mathrm{d}\epsilon_m/\mathrm{d}x. \tag{A 5}$$

The distances L_1, L_2 and L_3 are then given by

$$L_1 = \left(\frac{B''}{\mathrm{d}\epsilon_f/\mathrm{d}x+\mathrm{d}\epsilon_m/\mathrm{d}x}\right)^{\frac{1}{2}} \tag{A6}$$

from equations (A 3) and (A 4).

$$L_2 = L_1 + (\epsilon'_{mu}-\epsilon_r)\, L_3/\epsilon'_{mu}. \tag{A 7}$$

L_2 defines the outer edge of the zone of relaxation when fibres are present, see figure A 1 (*a*).

$$L_3 = 3(c^2-y^2)^{\frac{1}{2}}. \tag{A 8}$$

The system is analysed by dividing the length of the flaw into segments of width, dy, and unit thickness, where the length of the flaw, $2c$, is given by

$$c = g/E_{\mathrm{m}}\pi\epsilon_{\mathrm{mu}}^2, \tag{A 9}$$

i.e. assuming plane stress.

The first operation is to calculate the flaw opening such that the total extension of position L_2 is the same both inside and outside the relaxation zone, assuming the flaw to be fully closed in the unstrained condition.

The decrease in elastic strain energy (U_r) due to the presence of the flaw within each segment is given thus:

$$U_r = [\tfrac{1}{2}E_{\mathrm{c}}(\epsilon'_{\mathrm{mu}})^2 L_2 - \tfrac{1}{6}E_{\mathrm{m}}V_{\mathrm{m}}\epsilon_r^2 L_1 - \tfrac{1}{6}E_{\mathrm{f}}V_{\mathrm{f}}(\epsilon_{\mathrm{f,\,max}}^2 + \epsilon_{\mathrm{f,\,max}}\epsilon_r + \epsilon_r^2)L_1 - \tfrac{1}{2}E_{\mathrm{c}}(\epsilon'_{\mathrm{mu}})^2(L + L^3/3L_3^2 - L^2/L_3)] \times \delta y, \tag{A 10}$$

where $L = L_2 - L_1$.

The total decrease in elastic strain energy is obtained by numerically integrating equation (A 10) along the flaw, i.e. integrating y between the limits $-c \leqslant y \leqslant c$. This is repeated for a slightly larger value of c and then the rate of release of energy is obtained by dividing the difference between the two energies by the flaw extension.

Work is also done by friction, U_{f}, resulting from the relative displacement of fibre and matrix, so

$$U_{\mathrm{f}} = \frac{V_{\mathrm{f}}\tau\epsilon_{\mathrm{f,\,max}}L_1^2}{3r}\delta y. \tag{A 11}$$

A similar integration is performed as before, and to the subsequent rate of energy absorption is added the rate at which the matrix surfaces of the flaw absorb energy, i.e. gV_{m}.

The flaw will propagate catastrophically as soon as the rate of energy release becomes infinitesimally greater than the rate of absorption. In addition to the physical properties of fibre and matrix the computer requires an initial composite strain to examine for stability. Should the flaw be stable then the flaw is reset to its Griffith length and the process repeated with an increased strain.

The model has been shown to yield a satisfactory correlation with experimental data for polyalkene cements (see figure 6). It is predicted that the size of the relaxation zone is reduced by the presence of fibres, from a maximum 6.15 mm for the unreinforced matrix to 3.58 mm at 25 % (by volume) for the particular film considered. The maximum width of the critical flaw is similarly reduced from 1.43 μm to 1.04 μm respectively. These predictions are important since, if the model can be refined further such that (i) the length of the relaxation zone, either side of the flaw, is within the minimum crack spacing (equation (3)) and (ii) the model can allow for the numerous closely spaced flaws that are known to exist, it may be possible to fit a model, considering the growth of flaws, into the general multiple cracking theory of Aveston *et al.* (1971). This cannot be done for the model of Korczynskyj *et al.* (1981) because of the assumed constant size of the relaxation zone.

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Discussion

N. McN. Alford (*I.C.I. Runcorn, U.K.*). The beams that Dr Hannant showed, which exhibited multiple cracking, were tested in tension. If the beam is fractured under flexural centre point loading does multiple cracking still occur throughout the volume of the specimen even though there is a higher stress concentration at the load point?

D. J. Hannant. If a beam shows, anywhere on its surface, a strain in excess of ϵ'_{mu} we expect that part of the surface to show multiple cracking. Whether or not this strain, remote from the centre point (in three point bending), exceeds ϵ'_{mu} depends on the exact form of the rising load deflexion curve and on the dimensions of the specimen.

J. E. Bailey (*University of Surrey, U.K.*). The difficulty of obtaining values for the failure strain in tension for cement tests was mentioned. Dr Higgins did some careful tensile tests on well prepared and strain gauged specimens (w/c ratio 0.3). These specimens gave a failure strain *ca.* 0.05 %. Probably, in fibre composites, the matrix would be less perfect and this value can be regarded as an upper limit.

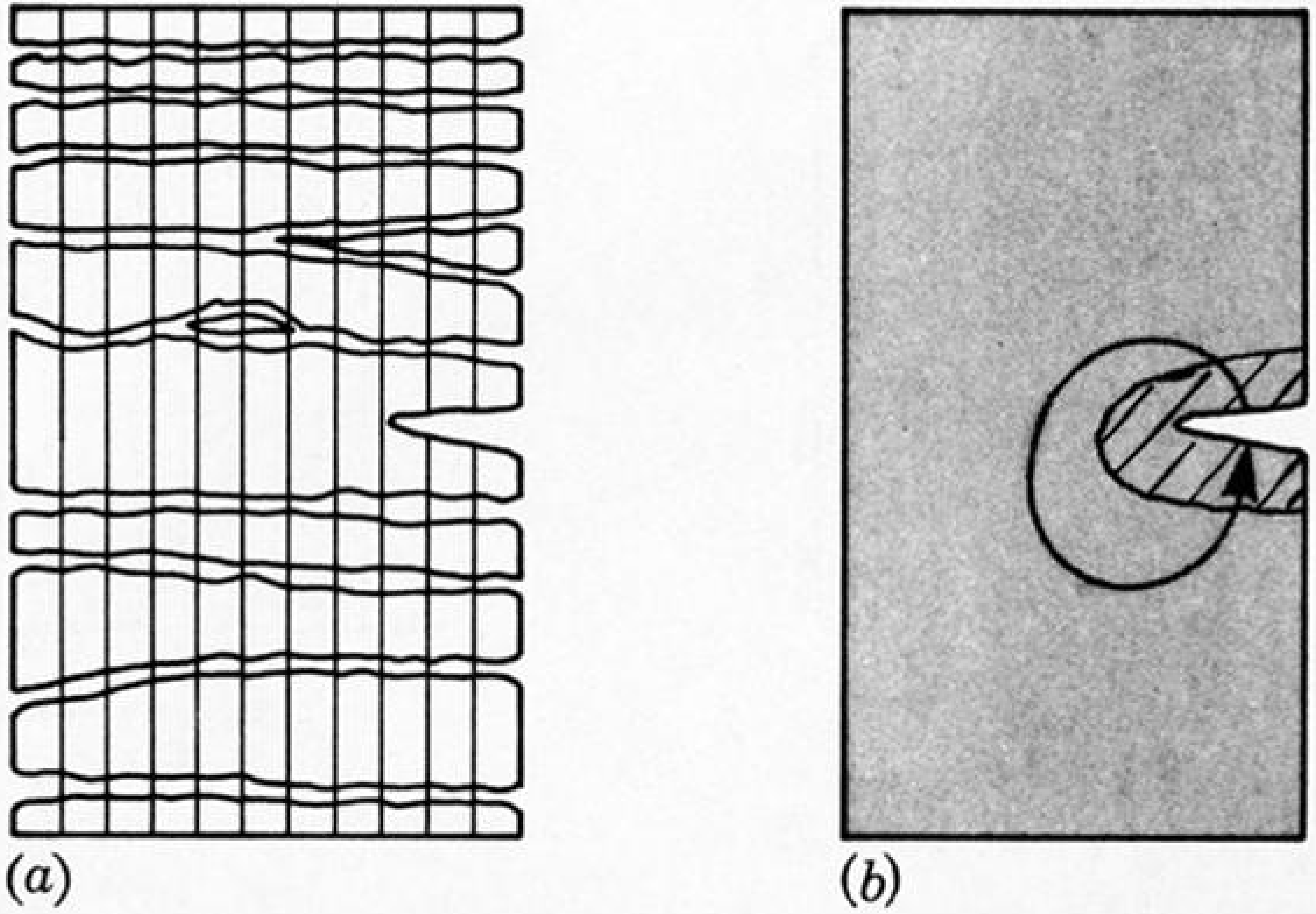
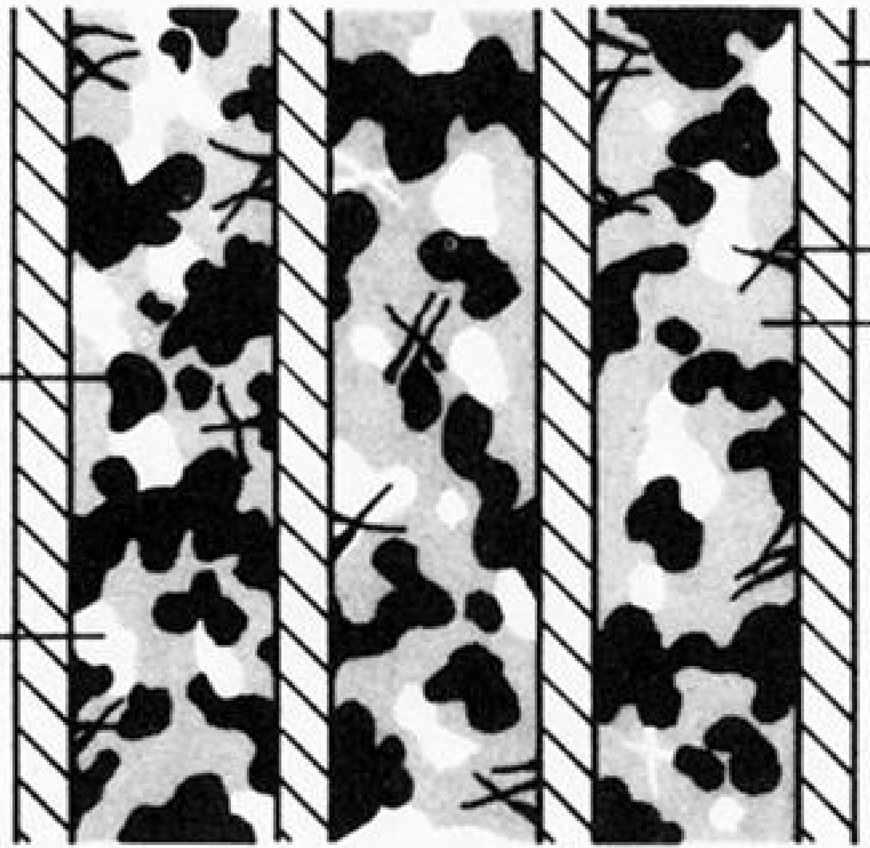


FIGURE 2. (*a*) A notched specimen containing fibres, which has undergone multiple fracture. (*b*) Model of a deformed notched specimen that is usually considered when one assesses fracture resistance with the *J*-integral.



IGURE 3. Schematic illustration of a cement matrix containing fibres. There are at least four phases present: fibres, unhydrated cement grains, hydration products and voids.